There are certain tests of divisibility that can help us to decide whether a given number is divisible by another number.

1. Divisibility of numbers by 2:
   ► A number that has 0, 2, 4, 6 or 8 in its ones place is divisible by 2.

2. Divisibility of numbers by 3
   ► A number is divisible by 3 if the sum of its digits is divisible by 3.

3. Divisibility of numbers by 4
   ► A number is divisible by 4 if the number formed by its last two digits (i.e. ones and tens) is divisible by 4.

4. Divisibility of numbers by 5
   ► A number that has either 0 or 5 in its ones place is divisible by 5.

5. Divisibility of numbers by 6:
   ► A number is divisible by 6 if that number is divisible by both 2 and 3.

6. Divisibility of numbers by 7:
   ► A number is divisible by 7, if the difference b/w twice the last digit and the no. formed by the other digits is either 0 or a multiple of 7. eg. 2975, it is observed that the last digit of 2975 is '5', so, 297 - (5x2) = 297 - 10 =287, which is a multiple of 7 hence, it is divisible by 7

7. Divisibility of numbers by 8:
   ► A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

8. Divisibility of numbers by 9:
   ► A number is divisible by 9 if the sum of its digits is divisible by 9.

9. Divisibility of numbers by 10:
   ► A number that has 0 in its ones place is divisible by 10.

10. Divisibility of numbers by 11:
    ► If the difference between the sum of the digits at the odd and even places in a given number is either 0 or a multiple of 11, then the given number is divisible by 11.

11. Divisibility of number by 12.
    ► Any number which is divisible by both 4 and 3, is also divisible by 12. To check the divisibility by 12, we i. First divide the last two-digit number by 4. If it is not divisible by 4, it is divisible by 4 is not divisible by 12. If it is divisible by 4 them. ii. Check whether the number is divisible by 3 or not.
    Ex: 135792 : 92 is divisible by 4 and also (1 + 3 + 5 + 7 + 9 +2 =) 27 is divisible by 3 ; hence the number is divisible by 12.

12. Divisibility by 13
    Oscillator for 13 is 4. But this time, our oscillator is not negative (as in case of 7) It is 'one-more' Oscillator. So, the working Principle will be different now.
    Eg: Is 143 divisible by 13 ? Sol: 14 3 : 14 + 3 x 4 = 26
    Since 26 is divisible by 13, the number 143 is also divisible by 13. Eg 2: Check the divisibility by 13. 2 416 7 26/6/20/34 [ 4 x 7 (from 24167 ) + 6 (from 24 167) = 34] [4 x 4 (from 3 4 ) + 3 (from 3 4 ) + 1 (from 24167)]
    =20 [4 x 0 (from 2 0 ) + 2 (from 20) + 4 (from 24 167)= 6]
    [4 x 6 (from 6 ) + 2 (from 24 167)= 26] Since 26 is divisible by 13 the number is also divisible by 13.

13. Divisibility by 14
    ► Any Number which is divisible by both 2 and 7, in also divisible by 14. That is, the number's last digit should be even and at the same time the number should be divisible by 7.
14. Divisibility by 15
► Any number which is divisible by both 3 and 5 is also divisible by 15.

15. Divisibility by 16
► Any number whose last 4 digit number is divisible by 16 is also divisible by 16.

16. Divisibility by 17
► Negative Oscillator for 17 is 5. The working for this is the same as in the case 7. Eg: check the divisibility of 1904 by 17
Sol: 190 - 5 x 4 = 170 Since 170 is divisible by 17, the given number is also divisible by 17. Eg 2: 957508 by 17
Sol: 95750 - 5 x 8 = 95710 9571 - 5 x 1 = 952 952 - 5 x 2 = 85 Since 85 is divisible by 17, the given number is divisible by 17.

17. Divisibility by 18
► Any number which is a divisible by 9 has its last digit (unit-digit) even or zero, is divisible by 18. Eg. 926568: Digit - Sum is a multiple of nine (i.e. divisible by 9) and unit digit (8) is even, hence the number is divisible by 18.

18. Divisibility by 19
► If recall, the ‘one-more’ osculator for 19 is 2. The method is similar to that of 13, which is well known to us. Eg 1 4 9 2 6 4 19/9/12/11/14

**General rules of divisibility for all numbers:**

- If a number is divisible by another number, then it is also divisible by all the factors of the other number.
- If two numbers are divisible by another number, then their sum and difference is also divisible by the other number.
- If a number is divisible by two co-prime numbers, then it is also divisible by the product of the two co-prime numbers.

**INTEGERS**

- Whole numbers are represented on the number line as shown here:

  ![Number Line](image)

- If you move towards the right from the zero mark on the number line, the value of the numbers increases.
- If you move towards the left from the zero mark on the number line, the value of the numbers decreases.

i. Negative integers: The numbers -1, -2, -3, -4... which are called negative numbers.

ii. Positive integers: The number 1, 2, 3, 4 ..s, which are called positive numbers.

- Euclid’s division lemma can be used to: a = b x q + r
- Find the highest common factor of any two positive integers and to show the common properties of numbers.

- Finding H.C.F using Euclid’s division lemma.
- Suppose, we have two positive integers ‘a’ and ‘b’ such that ‘a’ is greater than ‘b’. Apply Euclid’s division lemma to the given integers ‘a’ and ‘b’ to find two whole numbers ‘q’ and ‘r’ such that, ‘a’ is equal to ‘b’ multiplied by ‘q’ plus ‘r’.
- Check the value of ‘r’:
  - If ‘r’ is equal to zero then ‘b’ is the H.C.F of the given numbers.
  - If ‘r’ is not equal to zero, apply Euclid’s division lemma to the new divisor ‘b’ and remainder ‘r’. Continue this process till the remainder ‘r’ becomes zero. The value of the divisor ‘b’ in that case is the H.C.F of the two given numbers.
- Euclid’s division algorithm can also be used to find some common properties of numbers.

**Some Rules on Counting Numbers**

i. Sum of all the first n natural numbers = \(\frac{n(n+1)}{2}\)
   For eg.: 1 + 2 + 3 + .... + 105 = \(\frac{105(105+1)}{2}\) = 5565

ii. Sum of first n odd numbers = n²
   Eg: 1 + 3 + 5 + 7 = 4² = 16 (as there are four odd numbers)
   Eg: 1 + 3 +5 + +20th odd numbers
   (ie. 20 x 2 - 1 = 39 ) = 20² = 400

iii. Sum, of first n even numbers = n(n + 1)
   Eg: 2 + 4 +6 +8 + +100 (or 50th Even number)
   = 50 x (50 + 1 )= 2550

iv. Sum of Squares of first n natural numbers = \(\frac{n(n+1)(2n+1)}{6}\)
   For eg : 1² + 2² + 3² + .........10² = \(\frac{10(10+1)(2x10+1)}{6}\) = \(\frac{10 x 11 x 21}{6}\) = 385

v. Sum of cubes of first n Natural numbers = \(\left[\frac{n(n+1)}{2}\right]^2\)
   For eg : 1³ + 2³ + + ......... 6³
Square number:
In mathematics, a square number or perfect square is an integer that is the square of an integer; in other words, it is the product of some integer with itself. For example, 9 is a square number, since it can be written as 3 x 3.

Some Important Points
1. A number n is a perfect square if n = m^2 for some integer m
2. A perfect square number is never negative
3. A square number never ends in 2, 3, 7 or 8
4. The number of zeros at the end of the perfect square is even.
5. The square of an even number is odd and odd number is even.
6. A perfect square number never leaves remainder 0 or 1 on division by 3.
7. If a number has a square root then its unit digit must be 0, 1, 4, 5, 6, or 9.

Square Root of 1st Ten Number
\[ \sqrt{1} = 1 \quad \sqrt{2} = 1.41421 \ldots \quad \sqrt{3} = 1.73205 \]
\[ \sqrt{4} = 2 \quad \sqrt{5} = 2.23607 \ldots \quad \sqrt{6} = 2.44949 \]
\[ \sqrt{7} = 2.64575 \quad \sqrt{8} = 2.82842 \quad \sqrt{9} = 3 \]
\[ \sqrt{10} = 3.16227 \ldots \]

Square Root
\[ \sqrt{1} = 1 \quad \sqrt{4} = 2 \quad \sqrt{9} = 3 \quad \sqrt{16} = 4 \]
\[ \sqrt{25} = 5 \quad \sqrt{36} = 6 \quad \sqrt{49} = 7 \quad \sqrt{64} = 8 \]
\[ \sqrt{81} = 9 \quad \sqrt{100} = 10 \quad \sqrt{121} = 11 \quad \sqrt{144} = 12 \]
\[ \sqrt{169} = 13 \quad \sqrt{196} = 14 \quad \sqrt{225} = 15 \quad \sqrt{256} = 16 \]
\[ \sqrt{289} = 17 \quad \sqrt{324} = 18 \quad \sqrt{361} = 19 \quad \sqrt{400} = 20 \]
\[ \sqrt{441} = 21 \quad \sqrt{484} = 22 \quad \sqrt{529} = 23 \quad \sqrt{576} = 24 \]
\[ \sqrt{625} = 25 \quad \sqrt{676} = 26 \quad \sqrt{729} = 27 \quad \sqrt{784} = 28 \]
\[ \sqrt{841} = 29 \quad \sqrt{900} = 30 \quad \sqrt{961} = 31 \quad \sqrt{1024} = 32 \]
\[ \sqrt{1089} = 33 \quad \sqrt{1156} = 34 \quad \sqrt{1225} = 35 \quad \sqrt{1296} = 36 \]
\[ \sqrt{1369} = 37 \quad \sqrt{1444} = 38 \quad \sqrt{1521} = 39 \quad \sqrt{1600} = 40 \]
\[ \sqrt{1681} = 41 \quad \sqrt{1764} = 42 \quad \sqrt{1849} = 43 \quad \sqrt{1936} = 44 \]
\[ \sqrt{2025} = 45 \quad \sqrt{2116} = 46 \quad \sqrt{2209} = 47 \quad \sqrt{2304} = 48 \]
\[ \sqrt{2401} = 49 \quad \sqrt{2500} = 50 \]

Unit digit of square
<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit digit of square root</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

8. For every natural number n, \((n + 1)^2 - n^2 = (n + 1) + n\)
   For example: \(9^2 - 8^2 = 9 + 8\)
   \(81 - 64 = 17\)

9. The square of natural number n is equal to the sum of first n odd natural numbers
   \(Eg: 1 = 1^2 \quad 1 + 3 = 2^2 \quad 1 + 3 + 5 = 3^2\)

10. For any natural number m > 1, \((2m, m^2 - 1, m^2 + 1)\) is a Pythagoras triplet

Average
An average or more accurately an arithmetic mean is, in crude terms, the sum of n different data divided by n. For example, if a batsman scores 35, 45 and 37 runs in first, second and third innings respectively, then his average runs in 3 innings is equal to \(\frac{35 + 45 + 37}{3} = 39\) runs.
Therefore, the two mostly used formulas in this chapter are:
Average = \(\frac{\text{Total of data}}{\text{Total of data}}\) or = \(\frac{\text{Sum of observation}}{\text{No. of observation}}\)
And, Total = Average \(\times\) No. of Data

Direct Formula
Age of entrant = New average + No. of old members \(\times\) Increase

Direct Formula:
Weights of new person = weight of removed person + No. of person \(\times\) increases in average

Direct Formula:
Number of passed candidates = \(\frac{\text{Total candidate (Total average – Failed average)}}{\text{Passed average – Failed average}}\)
And number of failed candidates = \(\frac{\text{Total candidates (Passed average – Total average)}}{\text{Passed average – Failed average}}\)

Average Related to speed

Theorem: If a person travels a distance at a speed of \(x\) km/hr and the same distance at a speed of \(y\) km/hr, then the average speed during the whole journey is given by \(\frac{2xy}{x+y}\)

If half of the journey is travelled at a speed of \(x\) km/hr and the next half at a speed of \(y\) km/hr, then average speed during the whole journey is \(\frac{2xy}{x+y}\) km/hr.

If a man goes to a certain place at a speed of \(x\) km/hr and returns to the original place at a speed of \(y\) km/hr, then
the average speed during up-and down journey is \( \frac{2xy}{x+y} \) km/hr.

**Theorem:** If a person travels three equal distances at a speed of \( x \) km/hr, \( y \) km/hr and \( z \) km/hr respectively, then the average speed the whole journey is \( \frac{3xyz}{xy+yz+zx} \) km/hr.

**Proof:** Let the three equal distance be \( A \) km.

Time taken at the speed of \( x \) km/hr = \( \frac{A}{x} \) hrs.

Time taken at the speed of \( y \) km/hr = \( \frac{A}{y} \) hrs.

Time taken at the speed of \( z \) km/hr = \( \frac{A}{z} \) hrs.

Total distance travelled in time = \( \frac{A}{x} + \frac{A}{y} + \frac{A}{z} = 3A \) km

Average speed during the whole journey = \( \frac{3A}{\frac{A}{x} + \frac{A}{y} + \frac{A}{z}} = \frac{3xyz}{xy+yz+zx} \) km/hr

**Unlike terms:** - Terms that do not contain the same power of the same variable are called unlike terms.

**Eg:** - 3x and 3y, 3x and 6x

**Factor theorem:** P(x) is a polynomial and is a real number, if, \( p(a) = 0 \) then \( x-a \) is a factor of \( P(x) \)

**BASIC ALGEBRAIC EXPRESSIONS**

We know that the algebraic identities of second degree and these identities can be used to factorise quadratic polynomials.

A polynomial is said to be cubic polynomial if its degree is three

The algebraic identities used in factorizing a third degree polynomials are:

- \( (a+b)^2 = a^2 + 2ab + b^2 \)
- \( (a+b)^2 = (a-b)^2 + 4ab \)
- \( (a-b)^2 = a^2 - 2ab + b^2 \)
- \( (a-b)^2 = (a+b)^2 - 4ab \)
- \( a^2 - b^2 = (a+b)(a-b) \)
- \( a^2 + b^2 = (a+b)^2 - 2ab \)
- \( a^2 + b^2 = (a-b)^2 + 2ab \)
- \( (x+a)(x+b) = x^2 + (a+b)x + ab \)
- \( (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \)
- \( (a+b)^3 = a^3 + b^3 + 3ab(a+b) \)
- \( (a-b)^3 = a^3 - b^3 - 3ab(a-b) \)
- \( a^3 + b^3 = (a+b)(a^2 - ab + b^2) \)
- \( a^3 - b^3 = (a-b)(a^2 + ab + b^2) \)
- \( a^2 + b^2 + c^2 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \)
- \( (x+a)(x+b) = x^2 + (a+b)x + ab \)
- \( (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \)
- \( a + b = \frac{a^2 + b^2}{a^2 - ab + b^2} \)
The term percent means 'for every hundred'. It can be defined as: "A fraction whose denominator is 100 is called percentage, and the numerator or the fraction is called the rate per cent."

### PERCENTAGE

**Percent**

The term percent means 'for every hundred'. If can be defined as: "A fraction whose denominator is 100 is called percentage, and the numerator or the fraction is called the rate per cent."

<table>
<thead>
<tr>
<th>Percentage value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} = 50% )</td>
</tr>
<tr>
<td>( \frac{1}{6} = 16\frac{2}{3}% )</td>
</tr>
<tr>
<td>( \frac{1}{10} = 10% )</td>
</tr>
</tbody>
</table>

---

### Some more points on Percentage

1. If two values are resp. \( x \% \) & \( y \% \) more than a third value, then the first is the
   \[ \frac{100 + x}{100 + y} \times 100\% \]
   of the second.

2. If \( A \) is \( x \% \) of \( C \) & \( B \) in \( y \% \) of \( C \), then
   \[ A = \frac{x}{y} \times 100\% \]
   of \( B \)

3. \( x \% \) of a quantity is taken by the first, \( y \% \) of the remaining is taken by the second & \( Z \% \) of the remaining is taken by third person, now, if \( A \) is left in the fund, then there was
   \[ A \times \frac{100 \times 100 \times 100}{(100 - x)(100 - y)(100 - z)} \]
   in the beginning

4. \( x \% \) of a quantity is added. Again \( y \% \) of the increased quantity is added. Again, \( Z \% \) of the increased quantity is added. Now it becomes \( A \), then the initial amount is given by
   \[ A \times \frac{100 \times 100 \times 100}{(100 + x)(100 + y)(100 + z)} \]

5. The Population of a town is \( P \). It increases by \( x \% \) during first year, decreases \( y \% \) during the second year and again increases by \( Z \% \) during the third year. The population after third year will be
   \[ P \times \frac{(100 + x)(100 - y)(100 + z)}{100 \times 100 \times 100} \]

6. When the population decreases by \( y \% \) during the second year, while for the first and third years, it follows the same, the population after 3 years will be
   \[ p(100 + x)(100 - y)(100 + z) \]
   \[ 100 \times 100 \times 100 \]

7. If the Price of a commodity increases by \( r \% \), then the reduction in consumption so as not to increase the expenditure, is
**Cost Price:** The amount paid to purchase an article or the price at which an article is made is known as its cost price.

**Selling Price:** The price at which an article is sold is known as its selling price.

**Profit:** If the selling price (S.P) of an article is greater than the cost price (C.P), the difference between the selling price and cost price is called profit.

If S.P > C.P, then

Profit = S.P – C.P

**Loss:** If the selling price (S.P) of an article between the cost price (C.P) and the selling price (S.P) is called loss.

If C.P > S.P, then

Loss = C.P – S.P

Sell / sold/ Selling means S.P.
Buy/Cost/Purchase means C.P.

\[
S.P = \frac{C.P \times (100 + G\%)}{100}
\]

\[
S.P = \frac{C.P \times (100 - L\%)}{100}
\]

\[
C.P = \frac{S.P \times 100}{100 + G\%}
\]

\[
C.P = \frac{S.P \times 100}{100 - L\%}
\]
**DISCOUNT**

**Marked Price/List Price (M.P)**  
While buying goods that on every article there is a Price marked. This price is known as the marked price of the article.

**Discount**  
Sometimes shopkeepers offer a certain percent of rebate on the marked Price for cash payments. This rebate is known as discount.

Discount = MP – SP  
Rate of Discount (Discount%) = \[
\frac{\text{Discount}}{\text{MP}} \times 100
\]

\[
\text{SP} = \frac{\text{MP} \times \text{Discount} \%}{100}
\]

\[
\text{SP} = \text{MP} \times \left[1 - \frac{\text{Discount} \%}{100}\right]
\]

\[
\text{MP} = \frac{100 \times \text{SP}}{100 - \text{Discount} \%}
\]

More formulas based on different possibility
- % gain = \[
\frac{\text{True Value} - \text{False Value}}{\text{False Value}} \times 100
\]

- Total percentage profit = \[
\left(\frac{\% \text{ profit} + \% \text{ loss in weight}}{100 - \% \text{ Loss in weight}}\right) \times 100
\]

- Cost = \[
\text{Difference in percentage profit}
\]

- When cost Price & selling price are reduced by the same amount (Say A) then \[
\text{Cost Price} = \left(\frac{\text{initial profit} + \% \text{ increase in profit}}{100} \right) \times A
\]

- Selling Price = \[
\frac{\% \text{ gain} \times \% \text{ loss}}{100 - \% \text{ gain} - \% \text{ loss}}
\]

- Percentage Profit = \[
\frac{\text{first part} \times \% \text{ profit on first part} + \text{second part} \times \% \text{ profit on second part}}{\text{Total of two part}}
\]

- A man purchases a certain no. of articles at x a rupee and the same no at y a rupee. He mixes them together and sells them at z a rupee. Then his gain or loss% = \[
\frac{\text{p} \times 100}{\text{x} + \text{ny} + \text{pz}}
\]

**TIME AND WORK**

1. More men less days and conversely more days less men.
2. More men more work and conversely more work men.
3. More days more work and conversely more work days.

Also include the working hours \((T_1, T_2)\) for the two groups, then the relationship is:

\[
M_1 \times D_1 \times W_1 = M_2 \times D_2 \times W_2
\]

If \(M_1, W_1, D_1, W_1\) then \(M_2, W_2, D_2, W_2\)

\[
M_1 \times D_1 \times W_1 = M_2 \times D_2 \times W_2
\]

\[
5 \times 6 \times 6 = 2 \times D_2 \times 8 \times 10
\]

\[
D_2 = \frac{5 \times 6 \times 6}{12 \times 8 \times 10} = 3 \text{ days}
\]

2. If A can do a piece of work in \(x\) days and B can do it in \(y\) days then A and B working together will do the same work in \(\frac{xy}{x+y}\) days.
3. If A and B together can do a piece of work in \(x\) days and A alone can do it in \(y\) days, then B alone can do the work in \(\frac{xy}{x-y}\) days.

\[
\text{If A, B, & C can do a work in } x, y \text{ & } z \text{ days respectively, then all of them working together can finish the work in } \frac{xyz}{xy + yz + xz} \text{ days}
\]
**WORK & WAGES**

Wages are distributed in proportion to the work done and in indirect (or inverse) proportion to the time taken by the individual.

There are two methods:

Eg. A can do a work in 6 days and B can do the same work in 5 days. The contract for the work is Rs. 220. How much shall B get if both of them work together.

**Methods I:**

A's 1 day's work = $\frac{1}{6}$, B's 1 day's work = $\frac{1}{5}$

Ratio of their wages = $\frac{1}{6} : \frac{1}{5} = 5 : 6$

B's share = $\frac{220}{5 + 6} \times 6 = Rs. 120$

**Methods II:**

As wages are distributed in inverse proportion of number of days, their share should be in the ratio 5 : 6

B's share = $\frac{220}{11} \times 6 = Rs. 120$

---

**PIPES AND CISTERNS**

**Pipes & Cisterns:** These problems are almost the same as those of Time and work problems. Thus, if a pipe fills a tank in 6 hrs, then the pipe fills 1/6th of the tank in hour.

There is one difference that pipes & cisterns problems is that there are outlets as well as inlets. Thus, there are agents (the outlets) which perform negative work too. The rest of the process is almost similar.

**Inlet**

A pipe connected with a tank (or a cistern or a reservoir) is called an inlet, if it fills it.

**Outlet**

A pipe connected with a tank is called an outlet, if it empties it.

**Formulae**

**I.** If a pipe can fill a tank in x hours, then the part filled in 1 hour = $\frac{1}{x}$

**II.** If a pipe can empty a tank in y hours, then the part of the full tank emptied in 1 hour = $\frac{1}{y}$

**III.** If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened = \left( \frac{1}{x} - \frac{1}{y} \right)

**IV.** If a pipe can fill a tank in x hours and another can fill the same tank in y hrs, then the net part filled in 1 hr, when both the pipes are opened = \left( \frac{1}{x} + \frac{1}{y} \right)

**V.** If a pipe fills a tank in x hrs & another fills the same tank in y hrs, but a third one empties the full tank in z hrs, and all of them are opened together, the net part filled in 1 hr = $\left[ \frac{1}{x} + \frac{1}{y} - \frac{1}{z} \right]$

**VI.** A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank = $\frac{xy}{y-x}$ hrs.

Eg. Two pipes A & B can fill a tank in 45 hrs & 36 hrs. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol. Part filled by A alone in 1 hr = $\frac{1}{45}$

Part filled by B alone in 1 hr = $\frac{1}{36}$

Part filled by (A+B) in 1 hr = $\left( \frac{1}{45} + \frac{1}{36} \right)$ = $\frac{9}{180} = \frac{1}{20}$

Hence, both the pipes together will fill the tank in 20 hours.

---

**TIME AND DISTANCE**

**i.** Speed = \frac{Distance}{Time}

**ii.** Time = \frac{Distance}{Speed}

**iii.** Distance = Speed x Time (where D=distance, S=Speed, T=Time)

**iv.** x Km/hr = $\left( x \times \frac{5}{18} \right)$ km/hr

**v.** x metres/sec = $\left( x \times \frac{18}{5} \right)$ km/hr

**vi.** If speed of a body is changed in the ratio a : b, then the ratio of the time changes in the ratio b : a

**vii.** If a certain distance is covered at x km/hr and the same distance is covered at y km/hr then the average speed during the whole journey is $\frac{2xy}{x+y}$ km/hr

**viii.** If two persons A & B start at the same time in opposite directions from two points and after passing each other they complete the journey in ‘a’ & ‘b’ hrs, then

A’s speed: B’s Speed = $\sqrt{b} : \sqrt{a}$

**Some Direct Formulas:**

1. Required distance = \frac{Product of two speed}{Difference of two speed} x Difference b/w arrival times

2. Required distance = Total time taken x Product of the two speed / Addition of the two speed
3. If \( \frac{2}{3} \) rd of the distance is covered at \( x \) km/hr next \( \frac{2}{3} \) rd is covered at \( y \) km/hr and next \( \frac{1}{3} \) rd is covered at \( z \) km/hr then the average speed is km/hr.

10. If a train crosses a pole means it crosses it's own length.

11. If a train crosses a bridge means it crosses (it's own length + length of the bridge).

12. When a train is passing another train completely (whether moving in the same direction or in opposite directions), it has to cover a distance equal to the sum of the lengths of the two trains.

13. If two moving trains (in the same direction) crosses each other then relative speed = (speed of the faster trains—speed of the slower train)

14. If two trains are moving in the opposite directions then the relative speed = (speed of the 1st train + Speed of the 2nd train)

15. If without stoppage a train covers a distance at an average speed of \( x_1 \) km/hr and with stoppage it covers a distance at an average speed of \( x_2 \) km/hr then it's stoppage time per hour is \( t_2 = \frac{x_2 - x_1}{x_1} \) where \( t_2 \) = stoppage time per hour.

**Some Important points based on Trains**

1. When two trains are moving in opposite directions their speeds should be added to find the relative speed.

2. When they are moving in the same direction the relative speed is the difference of their speeds.

## Direct formulas based on Trains

**Upstream**:
- If the boat moves against the stream then it is called upstream.

**Downstream**: 
- If it moves with the stream, it is called downstream.

**Note**: If the speed of the boat (or the swimmer) is \( x \) and the speed of the stream is \( y \), then:

1. While upstream the effective speed of the boat = \( x - y \)
2. While downstream the effective speed of the boat = \( x + y \)

### Theorems based on streams (upstream & downstream)

1. If \( x \) km per hour be the man's rate in still water, & \( y \) km per hour be the rate of the current. Then

   \[
   x + y = \text{man's rate with current.}
   \]

   \[
   x - y = \text{man's rate against current.}
   \]

Adding & Subtracting and then dividing by 2

- \( x = \frac{1}{2} \) (man's rate with current + his rate against current)
- \( y = \frac{1}{2} \) (man's rate with current - his rate against current)
Facts:
1. A man’s rate in still water is half the sum of his rates with and against the current.
2. The rate of the current is half the difference between the rates of the man with and against the current.

Theorems:
1. A man can row x km/hr is still water. If in a stream which is flowing at y km/hr, it takes him z hrs to row to a place and back, the distance between the two places is \( \frac{z(x^2 + y^2)}{2x} \).
2. A man rows a certain distance downstream in x hours and returns the same distance in y hrs. If the stream flows at the rate of z km/hr then the speed of the man in still water is given by \( \frac{z(x + y)}{y - x} \) km/hr.

SIMPLE AND COMPOUND INTEREST

\[
\text{CI} = P \left(1 + \frac{r}{100}\right)^t
\]

or, \( A = P \left[1 + \frac{R}{100}\right]^n \)

When interest is compounded Half-yearly:

\[
\text{Amount} = P \left[1 + \frac{r}{200}\right]^{2t} = P \left[1 + \frac{r}{200}\right]^{2t}
\]

When interest is compounded quarterly:

\[
\text{Amount} : P \left[1 + \frac{r}{400}\right]^{4t} = P \left[1 + \frac{r}{400}\right]^{4t}
\]

When rate of interest is \( r_1 \%, r_2 \% \) & \( r_3 \% \) for 1st year, 2nd year & 3rd year,

\[
\text{Amount} = P \left[1 + \frac{r_1}{100}\right] \times \left[1 + \frac{r_2}{100}\right] \times \left[1 + \frac{r_3}{100}\right]
\]

If certain sum becomes ‘m’ times in ‘t’ years, the rate of compound interest

\[
r = 100 \left[ \left(m^{\frac{1}{t}} - 1 \right) \right]
\]

S.I. = \( \frac{100rt}{100 \left[1 + \frac{r}{100}\right]^t - 1} \)

\[
\text{Difference} = \frac{\text{sum} \times (100)^2}{r^2(300 + r)}
\]

Where, \( \text{Sum} = S \)

\[
\text{Amount} = P \left[1 + \frac{r}{100}\right]^t
\]

\[
\text{Difference} = \frac{S r^2 (300 + r)}{(100)^3}
\]
Pythagoras Theorem: - In a right angled the square of the hypotenuse is sum of the squares of the base of the perpendicular. $h^2 = p^2 + b^2$

A. Trigonometric Ratios

The ratios of the sides of a right - angled triangle with respect to its angles are called trigonometric ratios.

$AB = \text{Perpendicular (P)}$

$BC = \text{Base (B)}$ and $AC = \text{Hypotenuse (H)}$

Hint

Some People Have, Curly Black Hair, Turn Permentely Brown

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$0^\circ$</th>
<th>$30^\circ$</th>
<th>$45^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin\theta$</td>
<td>$\sqrt{1}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\cos\theta$</td>
<td>$1$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\tan\theta$</td>
<td>$0$</td>
<td>$\frac{\sqrt{3}}{3}$</td>
<td>$1$</td>
<td>$\sqrt{3}$</td>
<td>n.d.</td>
</tr>
</tbody>
</table>
D. Important Formula

1. \( \sin (A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B \)
2. \( \sin (A - B) = \sin A \cdot \cos B - \cos A \cdot \sin B \)
3. \( \cos (A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B \)
4. \( \cos (A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B \)
5. \( \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \)
6. \( \tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} \)
7. \( \cot (A + B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A} \)
8. \( \cot (A - B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A} \)
9. \( \tan (A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C}{1 - \tan A \cdot \tan B - \tan B \cdot \tan C - \tan C \cdot \tan A} \)
10. \( \sin (A + B) \cdot \sin (A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A \)
11. \( \cos (A + B) \cdot \cos (A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A \)
12. \( \sin 2A = 2 \sin A \cdot \cos A = \frac{2 \tan A}{1 + \tan^2 A} \)
13. \( \cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A} \)
14. \( \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \)
15. \( 2 \sin^2 A = 1 - \cos 2A \)
16. \( 2 \cos^2 A = 1 + \cos 2A \)
17. \( \sin 3A = 3 \sin A - 4 \sin^3 A \)
18. \( \cos 3A = 4 \cos^3 A - 3 \cos A \)
19. \( \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \)
20. \( \sin (A + B) + \sin (A - B) = 2 \sin A \cdot \cos B \)
21. \( \sin (A + B) - \sin (A - B) = 2 \cos A \cdot \cos B \)
22. \( \cos (A + B) + \cos (A - B) = 2 \cos A \cdot \cos B \)
23. \( \cos (A - B) - \cos (A + B) = 2 \sin A \cdot \sin B \)
24. \( \sin C + \sin D = 2 \sin \frac{C + D}{2} \cdot \cos \frac{C - D}{2} \)
25. \( \sin C - \sin D = 2 \cos \frac{C + D}{2} \cdot \sin \frac{C - D}{2} \)
26. \( \cos C + \cos D = 2 \cos \frac{C + D}{2} \cdot \cos \frac{C - D}{2} \)
27. \( \cos C - \cos D = -2 \sin \frac{C + D}{2} \cdot \sin \frac{C - D}{2} \)
28. \( \sin^{-1} x = x, -1 \leq x \leq 1 \)
\( \sin^{-1} (\sin x) = x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \)
\( \cos^{-1} x = x, -1 \leq x \leq 1 \)
\( \tan^{-1} x = x, -\infty < x < \infty \)
\( \tan^{-1} (\tan x) = x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \) etc.
1. If the position of the object is above the position of the observation then the angle made by the line joining object and observing point with the horizontal line drawn at the observation point is called angle of elevation.

2. If the position of the object is below the position of the observation the angle made by the line joining object and observation point with the horizontal line drawn at the observation point is called angle of depression.

3. The angle of elevation the top of a tower, standing on a horizontal plane, from a point A is, After walking a distance ‘d’ metres towards the foot of the tower, the angle of elevation is found to be β

\[ \text{The height of the tower } h = \frac{d}{\cot \alpha \cot \beta} \]

Where \( AB = d \)

4. If the Points of observation A and B lie on either side of the tower, then height of the tower.

\[ h = \frac{d \sin \alpha \sin \beta}{\sin (\alpha + \beta)} \]

(or) \[ h = \frac{d}{\cot \alpha + \cot \beta} \]

5. The angles of elevation of the top of a tower from the bottom and top of a building of height ‘d’ metres are \( \beta \) and \( \alpha \) respectively. The height of the tower is

\[ h = \frac{d \sin \beta \cos \alpha}{\sin (\beta - \alpha)} \text{ (or) } h = \frac{d \cot \alpha}{\cot \alpha - \cot \beta} \]

6. The angle of elevation of a cloud from a height ‘d’ metres above the level of water in a lake is ‘a’ and the angle of depression of its image in the lake is

\[ h = \frac{d \sin (\beta + a)}{\sin (\beta - a)} \text{ (or) } h = \frac{d (\tan \beta + \tan a)}{(\tan \beta - \tan a)} \]

7. The angle of elevation of hill from a point A is ‘a’. After walking to some point B at a distance ‘a’ metres from A on a slope inclined at ‘γ’ to the horizon, the angle of elevation was found to be β

\[ \text{Height of the hill } h = \frac{a \sin \alpha \sin (\beta - \gamma)}{\sin (\beta - \alpha)} \]

8. A balloon is observed simultaneously from the three points A, B, C on a straight road directly beneath it. The angular elevation at B is twice that at A and the angular elevation at ‘C’ is thrice that at A. If \( AB = a \) and \( BC = b \) then the height of the balloon \( h \) in terms of \( a \) and \( b \) is,

\[ h = \frac{a}{2b} \sqrt{(3b-a)(a+b)} \]
9. A flag staff stands on the top of a tower of height \( h \) metres. If the tower and flag staff subtend equal angles at a distance ‘d’ metres from the foot of the tower, then the height the flag-staff in metres is:

\[
h = \frac{d^2 + h^2}{\sqrt{d^2 - h^2}}
\]

**MENSURATION**

1. **Rectangle**
   A Quadrilateral with opposite sides equal and all the four angles equal to 90°

   - Perimeter of Rectangle: \( 2(l + b) \)
   - Area of Rectangle: \( l \times b \)
   - Other formula based on Rectangle
     
     \[
h = \sqrt{l^2 + b^2}, \text{ where } L \text{ (length)}
     \]
     
     B (Breath)
     
     H (diagonal)

   - Length = \( \frac{Area}{Breadth} \), Breadth = \( \frac{Area}{Length} \)

   - \((\text{Diagonal})^2 = (\text{Length})^2 + (\text{Breadth})^2\)

2. **Square**

3. **Parallelogram**
   A quadrilateral with opposite side parallel of equal.

   - Perimeter of Parallelogram: \( 2(a+b) \)
   - Area of Parallelogram: \( a \times h \)
   - Others: \( \text{base} = \frac{\text{area}}{\text{height}} \)

**Three types of Triangle**

i) **Right angled triangle**: A triangle with one angle equal to 90°

ii) **Isosceles triangle**: A triangle with any two sides equal

\[
\text{Area} = \frac{b}{4} \sqrt{4a^2 - b^2}
\]

In an isosceles triangle, opposite angles are equal

\[
h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2} = \frac{1}{2} \sqrt{4a^2 - b^2}
\]

iii) **Equilateral triangle**: A triangle with all sides equal and all angles are equal.
7. Quadrilateral

![Quadrilateral Diagram]

- Perimeter of quadrilateral: \(2d + (h_1 + h_2)\)
- Area of quadrilateral: \(\frac{1}{2} \times d \times (h_1 + h_2)\)

8. Circle

![Circle Diagram]

- Perimeter of Circle: \(2\pi r\) or \(\pi D\) (where \(D = 2r\))
- Perimeter of Semi Circle: \(\pi r + 2r\)
- Area of Circle: \(\pi r^2\)
- Area of Semi Circle: \(\frac{1}{2} \pi r^2\)

For Circumference:
\[C = 2\pi r\] or \(\pi D\)
\[
r = \frac{D}{2} = \text{radius} = \text{OA} = \text{OB}
\]
\[
D = \text{diameter} = \text{AB}
\]
\[
CD = \text{Chord}
\]

9. i) ARC of a Circle

A portion of the Perimeter (or a part of the curved portion) of the circle.

- Length of Arc \(l = \frac{\theta}{180} \times \pi r\) or \(\frac{\theta}{360} \times 2\pi r\)

- \(\theta\) is the central angle in degrees.

- Arc \(AB = \text{Length of AB}\)

- \(\text{Arc } AB = \frac{2\pi \theta}{360}\), where \(\angle AOB = \theta\) and \(O\) is the center.
10. Regular Hexagon

- Perimeter of Regular Hexagon: $6 \times a$
- Area of Regular Hexagon: $\frac{3\sqrt{3}}{2} \times a^2$
- Others: Each interior angle
- $(2n-4) \times 90^\circ$  
  Sum of total interior angles = $(2n-4) \times 90^\circ$

12. Area of 4 walls of a room = $2 \times (l+b) \times h$ or Perimeter × Height

**Mensuration – II**

1. Cuboid:

Let length = $l$, breadth = $b$ and height = $h$ units

i. Volume of Cuboid = $(l \times b \times h)$ Cubic Units

$\therefore$ Sector of a circle: The area covered between an arc, the centre and two radii of the circle.

shaded portion = sector $AOB$

- Area of sector $AOB = \frac{\pi r^2 \theta}{360^\circ}$
- Area of Sector $AOB = \frac{1}{2} \times \text{arc} \ AB \times r$

iii. Segment of a circle:

- Area of Segment of a circle = $r^2 \left[ \frac{\pi \theta}{360^\circ} - \frac{\sin \theta}{2} \right]$
  or, $\frac{\pi r^2 \theta}{360^\circ} - \frac{r^2 \sin \theta}{2}$

(iv) Ring: $\pi (R^2 - r^2)$  

$R =$ Radius of bigger circle  
$r =$ Radius of smaller circle

**2. Cube**

Let each edge (or side) of a cube be a units. Then

i. Volume of the cube = $a^3$ cubic units
ii. Whole surface area of the cube = $(6a^2)$ sq. units
iii. Diagonal of the cube = $(\sqrt{3}a)$ units.

**3. Cylinder**

Let the radius of the base of a cylinder be $r$ units and its height (or length) be $h$ units Then:

i. Volume of the cylinder = $(\pi r^2 h)$ cu. units
ii. curved surface area of the cylinder = $(2 \pi rh)$ sq. units
iii. Total surface area of the cylinder = $(2 \pi rh + 2 \pi r^2)$ sq. units

**4. Sphere**

Let the radius of a sphere be $r$ units Then:

i. Volume of the sphere = $\left(\frac{4}{3} \pi r^3\right)$ cu. units
ii. Surface area of the sphere = $(4 \pi r^2)$ sq. units
5. Hemisphere:
   i) Volume of a hemisphere = \(\frac{2}{3} \pi r^3\) cu. units
   ii) Curved surface area of the hemisphere = \(2\pi r^2\) sq. units
   iii) Whole surface area of the hemisphere = \(3\pi r^2\) sq. units

6. Right Circular Cone:

   Let \(r\) be the radius of the base, \(h\) the height and \(l\) the slant height of a cone. Then:
   i) Slant height \(l = \sqrt{h^2 + r^2}\)
   ii) Volume of the cone = \(\frac{1}{3} \pi r^2 h\) cu. units
   iii) Curved Surface area of the cone = \(\pi r l\) sq. units
   iv) Total surface area of the cone = \(\pi r(l+r)\) or \(\pi rl + \pi r^2\)

7. Frustum of a right circular cone:
   If a cone is cut by a plane parallel to the base so as to divide the cone into two parts. The lower part is called frustum of the cone.

   Let the radius of the base of the frustum = \(R\), the radius of the top = \(r\), height = \(h\) & slant height = \(l\) units

8. Right Parallelopiped:

   - Curved surface area = \(\pi (r+R)l\) sq. units
   - Total surface area = \(\pi [(r+R)l + r^2 + R^2]\) sq. units
   - Volume = \(\frac{\pi h}{3} (r^2 + R^2 + rR)\) cu. units

---

**CIRCLE**

Definition: A circle is the locus of a point which moves so that its distance from a fixed point.

i. Equation of a circle:
   a. \((x - h)^2 + (y - k)^2 = a^2\)
   b. \(x^2 + y^2 = a^2\) for the origin

ii. General Equation: If centre is \((h, k)\) and radius \(a\) is

\[ (x - h)^2 + (y - k)^2 = a^2 \]

\[ x^2 + y^2 + 2gx + 2fy + c = 0 \]

where \(g = -h, f = -k\) and \(c = h^2 + k^2 - a^2\)

\[ \therefore \text{Centre is} (-g, -f) \text{and radius is} \sqrt{g^2 + f^2 - c} \]

iii. Conditions for the equation of a circle for the general from.

\[ x^2 + y^2 + 2gx + 2fy + c = 0 \]

multiply throughout by \(a\) (i.e.)

\[ ax^2 + ay^2 + 2gax + 2fay + ac = 0 \]

a. It should be an equation of the second degree in \(x\) and \(y\).

b. The co-efficients of \(x^2\) and \(y^2\) should be equal.

c. There should be no term involving the product \(xy\).

\[ \text{d. Diameter from} \ (x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0 \]
1. Theorem of $30^\circ-60^\circ-90^\circ$ triangle: If the angles of a triangle are $30^\circ, 60^\circ$ and $90^\circ$, then the side opposite to $30^\circ$ is half of the hypotenuse & the side opposite to $60^\circ$ is $\frac{\sqrt{3}}{2}$ times the hypotenuse.

![Diagram of a $30^\circ-60^\circ-90^\circ$ triangle]

i.e. $BC = \frac{1}{2} AC$, $AB = \frac{\sqrt{3}}{2} AC$

2. Theorem $45^\circ-45^\circ-90^\circ$ Triangle. If the angles of triangle are $45^\circ$, $45^\circ$, $90^\circ$ then the perpendicular side are $\frac{1}{\sqrt{2}}$ times the hypotenuse e.g. in the above figure.

$AB = BD = \frac{1}{\sqrt{2}} AD$

ALL The Best for upcoming examinations!!